

# ALGEBRAIC CHARACTERIZATIONS OF QUANTUM COMPUTATIONAL LOGICS

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Classical circuit theory is basically irreversible in the sense that Boolean functions (gates) are generally described as many-to-one. We know, however, that every Boolean gate has its own reversible counterpart, as shown by Toffoli ([7]). The classical circuit-model of computation, both in its reversible and in its irreversible version, can be formulated by using a very small set of gates, called *functionally universal set of gates*.

Quantum computation “originates” in a naturally reversible way, because quantum gates are interpreted as unitary operators acting on pure states (*qubits* or *quregisters*) of the Hilbert space associated with the quantum circuit at issue. Since there are uncountably many unitary operators, there is no hope to find any *finite* functionally universal set of quantum gates. The best we can do is having recourse to the notion of finite *approximate universality*.

Shi ([6]) and Aharonov ([1]) have shown that the Toffoli gate and the Hadamard gate give rise to an approximately universal set of quantum computational gates. We study the basic algebraic properties of this system by introducing the notion of *Shi-Aharonov quantum computational structure*. We show that the quotient of this structure is isomorphic to a structure based on a particular set of complex numbers (the closed disc with center  $(\frac{1}{2}, \frac{1}{2})$  and radius  $\frac{1}{2}$ ). On this basis, one can say that the “logic” of the Shi-Aharonov system of quantum computational gates is nothing but a complex-valued logic.

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