Topological quantum computation and quantum logic

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Microsoft Station Q

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Microsoft Project Q:

Search for non-abelian anyons in topological phases of matter, and build a topological quantum computer.

Theory:

MS station Q

Experiments:

Bell lab, Harvard, Columbia, U Chicago, Caltech, Princeton, Weizmann

Microsoft Station Q
http://stationq.ucsb.edu/

- Michael Freedman (math)
- Chetan Nayak (physics)
- Matthew Fisher (physics)
- Kevin Walker (math)
- Matthew Hastings (cs)
- Simon Trebst (computational physics)
- Parsa Bonderson (physics)

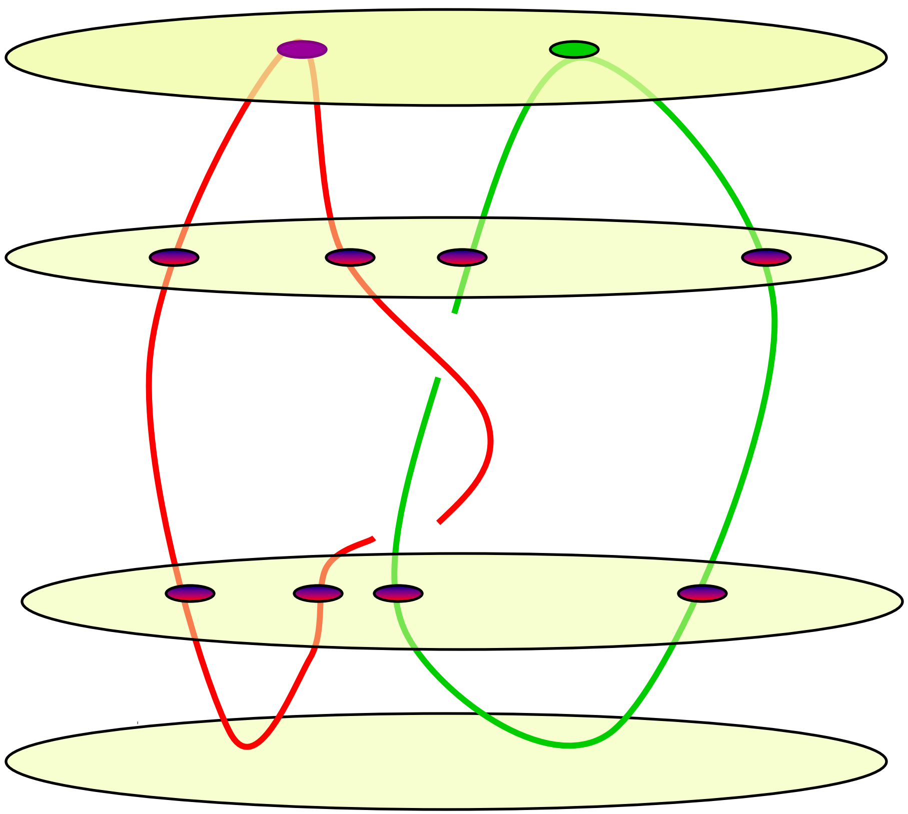
Topological Computation

Computation

output

apply operators

initialize



Physics

measure

braid

create particles

Statistics of Identical Particles

non-local or topological interaction

Given a collection of n identical particles in a space X, at each moment the state of the n particles is given by a wavefunction $|\psi>$,

Suppose at a later time, the n particles return to the same positions as a set, how does $|\psi>$ change?

Answers depend on dimensions of X.

Statistics of Particles

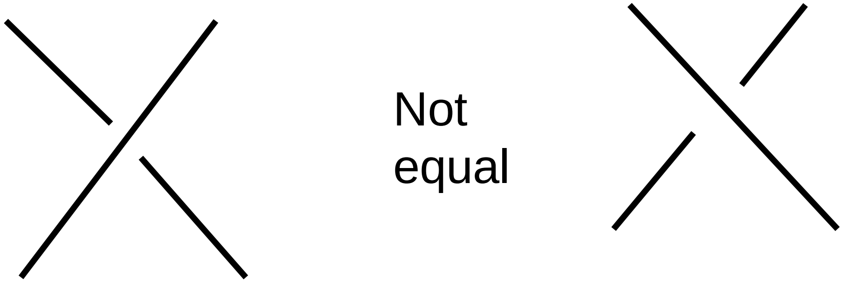
In R^3, particles are either bosons or fermions

Worldlines (curves in $R^3 \times R$) exchanging two identical particles depend only on permutations

Statisitcs is λ: $S_n \to Z_2$

Braid statistics

In R^2, an exchange is of infinite order



Braids form groups B_n

Statistics is λ: $B_n \to U(1)$

If not 1 or -1, but $e^{i\theta}$, anyons

Non-abelian anyons

Suppose the ground state of n identical particles is degenerate, and has a basis $\psi_1, \psi_2, \ldots, \psi_k$

Then after braiding some particles:

$\psi_1 \to a_{11}\psi_1 + a_{12}\psi_2 + \ldots + a_{k1}\psi_k$

.

Particle statistics is $\lambda: B_n \to U(k)$

Particles with k>1 are called non-abelian anyons

Topological phases of matter or anyonic quantum systems

A quantum system whose lowest energy states are effectively described by a topological quantum field theory (TQFT)

Given a theory, put it a surface Y,

Hilbert space H(Y)$\cong\oplus$ V_i(Y)---energy λ_i

Assume energy gap $\lambda_1 > \lambda_0$=0,

Y $\to$ V^{top}(Y) (part of V_0(Y)) is a TQFT

Some features

1) Ground states degeneracy---dim$V^{top} \geq 1$ (memory)
2) No non-trivial continuous evolutions
   (fault-tolerant or deaf)
3) Elementary excitations are “anyons”
   (braiding statistics are gates)

TQFT=Modular Tensor Cat

A ribbon category is a braided fusion category with compatible duality=charge conjugation, which yields link invariants such as Jones poly and representations of braid groups.

A ribbon tensor category with finitely many isomorphism classes of simple objects and a non-singular s-matrix.

Simple objects represent anyons.
Tensor product is fusion.

Non-abelain anyons, mathematically?

Are non-abelian anyons possible,

ie, are there unitary braid group representations?

Jones reps through Temperley-Lieb algebras labeled by r=3,4,5,… (1981)

Jones polynomials at r-th root of unity

---computationally hard if $r \neq 3,4,6$

Non-abelian anyons, physically?

- If there were non-abelian anyons, then they can be used to built universal fault-tolerant quantum computers
- Do they exist in Nature?
- There is evidence and numerical “proof” that they do exist in fractional quantum Hall liquids

Classical Hall effect

E. H. Hall, 1879

On a new action of the magnet on electric currents

Am. J. Math. Vol 2, No.3, 287--292

"It must be carefully remembered that the mechanical force which urges a conductor carrying across the lines of the magnetic force, acts, not on the electric current, but on the conductor which carries it"

Maxwell, Electricity and Magnetism

Quantum Hall Effect

1980 K. von Klitzing ---IQHE

(1985 Nobel)

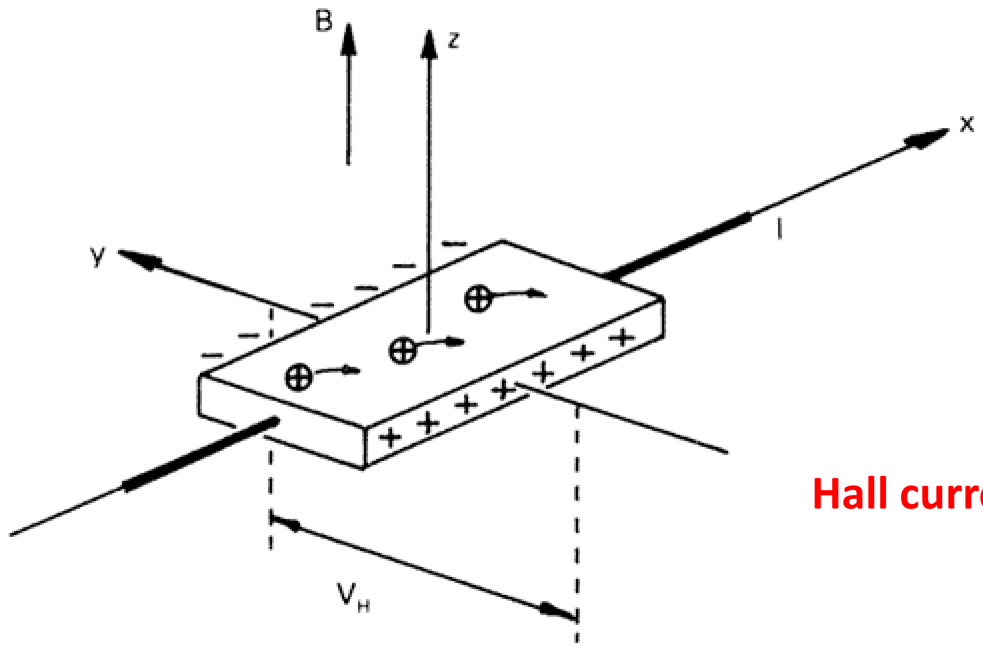
1982 H. Stormer, D. Tsui ---FQHE

R. Laughlin (1998 Nobel)

quasi-particle with 1/3 electron charge

and braiding statistics (anyons)

Electrons in a flatland

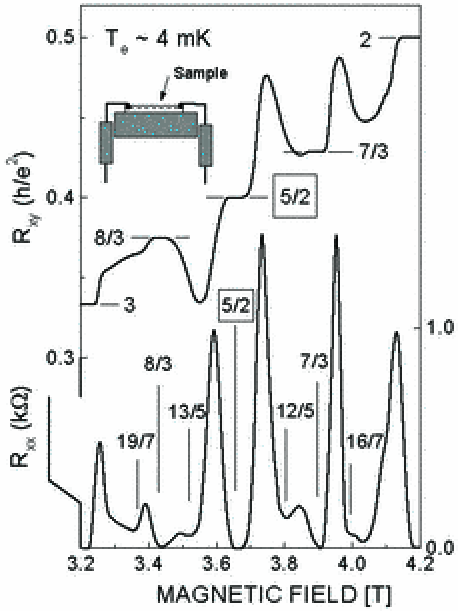


Hall current I_H

Energy levels for electrons are called Landau levels,
the filling fraction ν=# of electrons/# flux lines

Non-Abelian anyons in real life: FQHE?

Fig. 1, Pan et al



Read-Rezayi conjecture:

ν=1/3 or 2/3 $\longleftrightarrow$ **Jones rep at r=3**

ν=5/2 $\longleftrightarrow$ **Jones rep at r=4**

ν=12/5 or 13/5 $\longleftrightarrow$ **Jones rep at r=5**

(Universal QC)

Experimental Progress

- For $\nu=5/2$, the charge of e/4 particles is confirmed

- No conclusive experiments to prove any anyonic statistics, but progress has been made for the last 4 years (Goldman for abelian, and Willet for 5/2)

TQC to Quantum Logic?

Is it possible to address the “touchy and complicated” issue:

(von Neumann)

What is a physical proposition?

Quantum Logics

- Birkhoff-von Neumann (1936):

  Continous geometry

- 1960---1970’s:

  Orthomodular lattice

- Third life (Dunn): ?

Continuous Geometries (CGs)

A continuous geometry of von-Neumann:

orthcomplemented complete modular lattice (Kaplanski)

Is the word problem decidable in CGs?

In general, they should be very similar to quantum logics of finite dimensional vector spaces.

Qubit continuous geometry

- $PG(2^n)$=subspaces of n-qubits

  $PG(2^n)$ embeds isomorphically in $PG(2^{n+1})$

  $$p \in PG(2^n),\ \ p \to p \quad \mathbf{C}^2$$

- Normalized dimension $\delta(p)=d(p)/d(1)$, metrically completed by

  $$|p-q|=\delta(p \vee q)-\delta(p \wedge q)$$

Type II_1 factors

- A von Neumann algebra M is a unital *-algebra of bounded operators on Hilbert space H such that M=M”.

  M is a factor if its center $Z(M)=\mathbf{C}$
- A factor N is II_1 if it has a unique trace $tr: N \to \mathbf{C}$ s.t. {tr(p): p a projector}=[0,1].
- The lattice of projectors=lattice of invariant subspaces is a CG.

Qubit II_1 factor

- $M_2(\mathbf{C})$=all 2×2 matrices,
  inclusion of $M_2(\mathbf{C})$ to $M_4(\mathbf{C})$
  by $A\to A \quad \mathbf{I}$

- Define a normalized trace $tr(\mathbf{I})=1$, and then complete the union of $M_{2n}(\mathbf{C})$to a II_1 factor

Jones towers

Given II_1 factors $N \subset M$, Jones construct a tower

$N \subset M \subset M_2 \subset \ldots$

II_1 factor M_i ($M_0=N$, $M_1=M$) is obtained from M_{i-1} by adjoining a projector

e_i: $L^2(M_i,tr) \to L^2(M_{i-1},tr)$.

The e_i's form the Temperley-Lieb algebras.

Temperley-Lieb algebras

Fix d, $TL_n(d)$ is the finite dimensional algebra generated by 1, $e_1,\ldots,e_{n-1}$

$$e_i^2=e_i=e_i^*$$

$$e_ie_j=e_je_i \text{ if } |i-j|\geq 2$$

$$e_i\, e_{i\pm 1}\, e_i=1/d\, e_i$$

Geometry of TL algebras

- e_i's are projectors
- images of e_i and e_j are orthogonal modulo their intersection if $|i-j| \geq 2$
- "angle" between ith and (i+1)th are determined by d.

Jones Rep of the Braid Groups

The braid group B_n has a presentation:

$$\{1, \sigma_1, \ldots, \sigma_{n-1}\}$$

$$\sigma_i\sigma_j=\sigma_j\sigma_i \text{ if } |i-j| \geq 2$$

$$\sigma_i \sigma_{i+1}\sigma_i=\sigma_{i+1}\sigma_i\sigma_{i+1}$$

Fix $q=e^{2\pi i/r}$, Jones rep: $\sigma_i \to q-(1+q)e_i$

TQC to QL

Type II_1 factors are behind modular tensor categories describing statistics of nonabelian anyons in topological phases of matter, which are pursued as hardware for topological quantum computers.

It is also known Type II_1 factors are determined by their modular lattices.

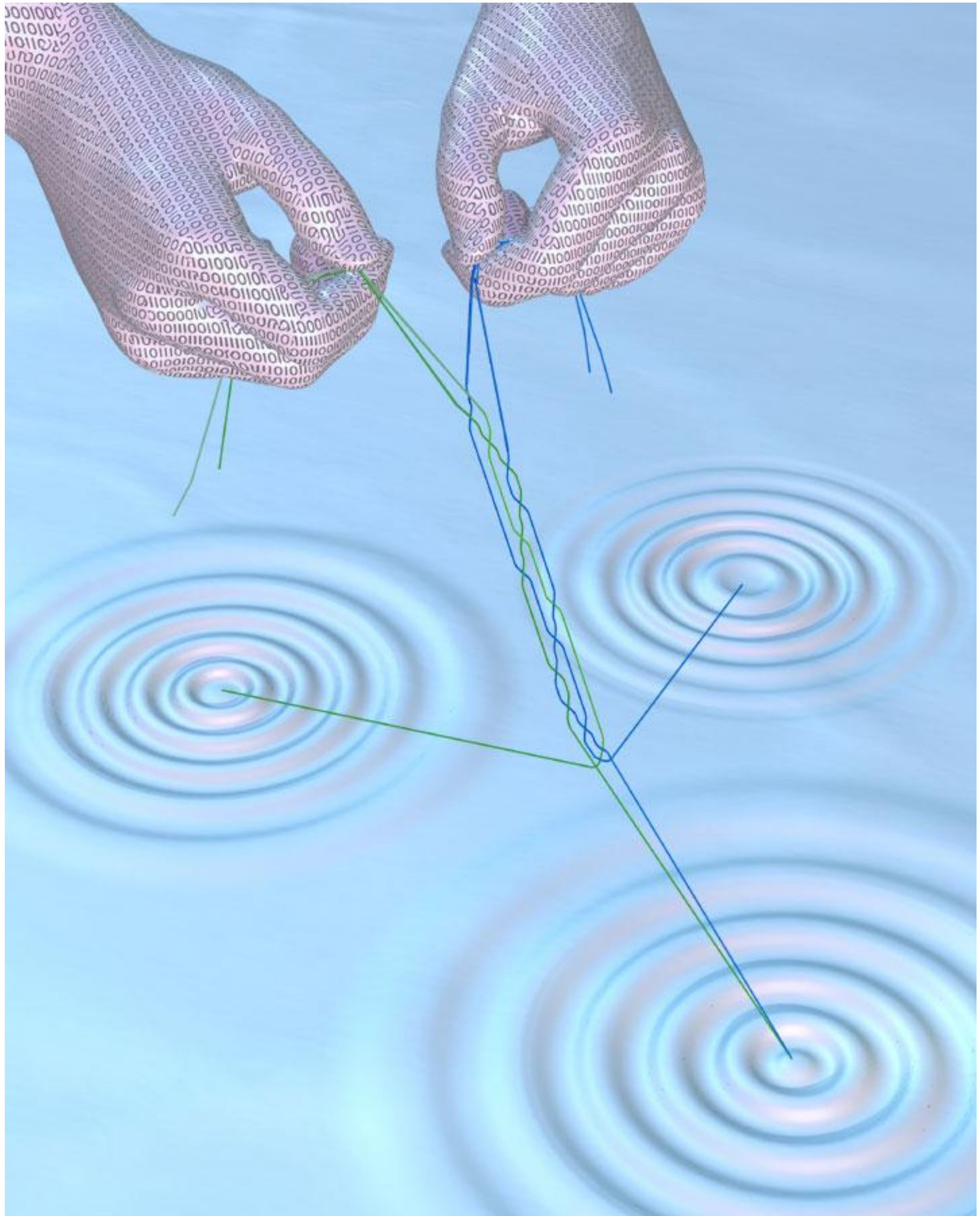
What can we learn about the "touchy and complicated" (von Neumann) issue through II_1 factors:

What is a physical proposition?

Can we axiomatize projectors of computable traces?

1. Can quantum logics help the construction of a universal quantum computer?

2. Will the interaction of quantum logics and quantum computation result in a more physical quantum framework?



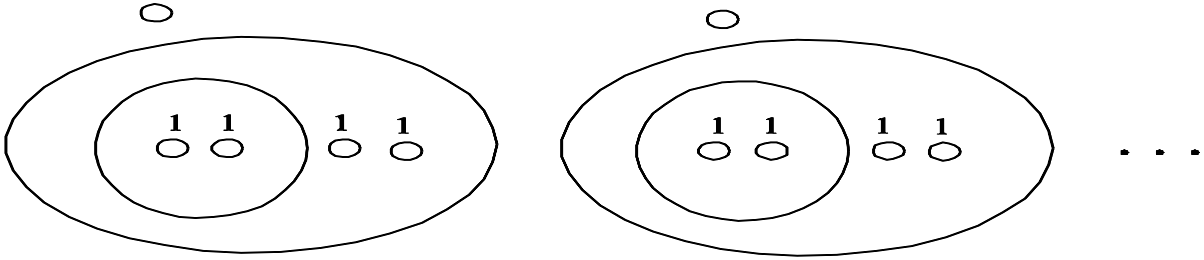
Topological models:

A topological model can be constructed using any Jones representation for any r:

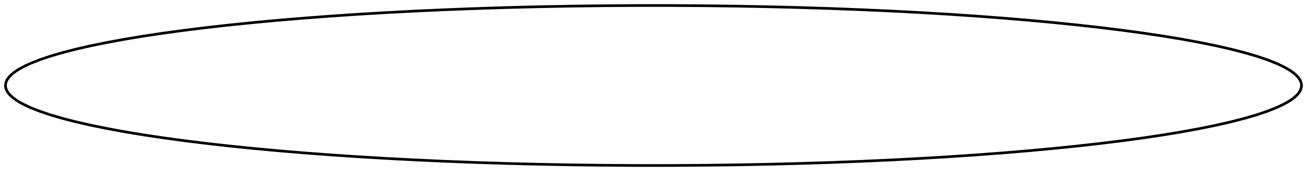
Fix r=5,

For 1-qubit gates, $\rho_5: B_4 \rightarrow U(2)$ or $U(3)$

For 2-qubits gates, $\rho_5: B_8 \rightarrow U(13)$ or $U(21)$



For n qubits, consider the 4n punctured disk D_{4n} and
$\rho_5: B_{4n} \to U(N_{4n})$



Given a quantum circuit on n qubits:

$$U_L: (C^2)^{\ n} \to (C^2)^{\ n}$$

Ideally to find a braid $b \in B_{4n}$ so that the following diagram commutes (almost FKW):

$$\begin{tikzcd}
(C^2)^{\ n} \arrow[r] \arrow[d, "U_L"'] & V(D_{4n}) \arrow[d, "\rho_{CS5}(b)"] \\
(C^2)^{\ n} \arrow[r] & V(D_{4n})
\end{tikzcd}$$